Fatigue crack propagation in unplasticized poly(vinyl chloride) (uPVC): 2. Near-threshold fatigue crack growth*

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A new computer-aided test method was developed to measure near-threshold fatigue crack growth rates for a 150 mm class 12 uPVC pipe. A computer program was written to drive the fatigue testing machine and an optical microscope of 0.01 mm resolution was used to determine crack growth in the single-edge-notched specimen. The efficiency and advantages of this new technique are discussed and compared to the current ASTM proposed method. For crack growth rates of the order of 10^{-9} m cycle^{-1} the present computerized method was approximately seven times more efficient and therefore greatly minimized the testing times required to collect near-threshold fatigue data. Using this method the effects of stress ratio and level of processing on the near-threshold fatigue crack growth were investigated. It is observed that multiple crazes were predominant at stress ratios larger than 0.6 but they apparently disappeared at lower stress ratio values. This phenomenon was explained in terms of a critical stress criterion for craze formation and the elastic stress field near the crack tip. The fatigue threshold ΔK_{th} at a crack growth rate of 5×10^{-10} m cycle^{-1} was found to decrease linearly with stress ratio from 0 to 0.5 and to remain constant thereafter. The two levels of processing studied for this uPVC pipe material did not produce any effect on ΔK_{th}.

(Keywords: poly(vinyl chloride); fatigue crack growth; near threshold; fatigue threshold; overloading; stress ratio; single and multiple crazes; processing effect)

INTRODUCTION

Fatigue has been identified as a common cause of failure in unplasticized poly(vinyl chloride) (uPVC) pipes, owing to crack propagation initiated from pre-existing flaws[1-3]. In design against this failure mode, it is important to determine the fatigue threshold stress intensity factor (ΔK_{th}) below which cracks do not propagate when they are subjected to cyclic loads. However, important as it is, a 'true' ΔK_{th} is difficult to measure since this requires very long testing times. Usually, near-threshold fatigue crack growth rates of less than 10^{-8} m cycle^{-1} are determined and then used to estimate ΔK_{th}. Even so, obtaining the near-threshold crack growth data is a tedious time-consuming procedure. It is, therefore, not surprising that relatively little work has been done on the fatigue threshold of uPVC pipes, let alone the effects of processing on ΔK_{th}. Perhaps the exception that we are aware of is that of Jilken and Gustafson[4]. They studied the effects of mean stress and processing on ΔK_{th}, but the frequencies used were 40 and 60 Hz, which were considered too high for realistic service conditions. In addition, at such high frequencies crack tip adiabatic heating must have occurred.

We have developed a new computer-aided test method to measure near-threshold fatigue crack growth rates for a uPVC pipe. The method has been developed primarily to minimize the testing times involved in obtaining the stress intensity factor range (ΔK) for a given crack growth rate (da/dN) at a constant stress ratio (R). It is shown that this method is much faster than the current proposed ASTM method for measuring fatigue crack growth data near the threshold[5]. Using the new testing technique we have studied the effects of mean stress and processing on the fatigue threshold of a uPVC pipe-grade material.

MEASUREMENT OF FATIGUE THRESHOLD AND NEAR-THRESHOLD FATIGUE CRACK GROWTH

In the past, both increasing and decreasing ΔK methods have been proposed to determine near-threshold fatigue data[6-11]. There have also been some attempts using high frequencies for materials where few frequency effects exist[12,13]. Unfortunately, this is unsuitable for polymers such as uPVC because they are sensitive to frequency variations.

There are two variations of the increasing ΔK method[7,10]. In one case[10] the applied load is shed as quickly as possible in the pre-cracking stage without causing crack arrest until the crack growth rate is about 10^{-11} m cycle^{-1}. The load amplitude is now maintained constant and, since in most specimen geometries ΔK increases as the crack length is increased, near-threshold fatigue data can be easily obtained. In the other case[7] the load is reduced dramatically, when the crack growth rate is approximately 10^{-9} m cycle^{-1}, to result in crack arrest. This low ΔK is maintained for 5×10^{6} cycles. If no crack growth occurs, the applied ΔK is increased in steps by 5% and kept for 5×10^{6} cycles until crack growth resumes. To ensure that there is no previous overloading effect the whole procedure is repeated once again until crack growth is measurable. These two forms of the increasing

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ΔK method are tedious and time-consuming. The increasing ΔK method has in fact given way to the now preferred decreasing ΔK method.

The conventional decreasing ΔK method makes use of a load shedding schedule where the load is reduced at selected crack length (a) intervals according to some form of equation. Load shedding can be achieved either incrementally by manual control or continuously by computerized automated control. Because of the ease of acquiring and processing raw data and hence the elimination of intensive manual labour the automated technique is preferred. However, this does rely on the ability to measure crack length positions continuously and accurately using one of the many methods already developed, e.g. compliance measurement and potential drop. This information is needed to input to the load shedding program to reduce the applied ΔK by appropriate amounts. Following the early work of Saxena *et al.*[14] the ASTM E24 Committee has recommended that the following load shedding equation be used:

$$\Delta K = \Delta K_i \exp[C(a - a_i)] \quad (1)$$

Here, (ΔK_i,a_i) and (ΔK,a) are initial and instantaneous values of applied stress intensities and crack lengths. The constant C has a physical dimension of $(\text{length})^{-1}$ given by[5]

$$C = \frac{1}{K}\frac{dK}{da} \leqslant 0.08\,\text{mm}^{-1} \quad (2)$$

A limit on C assumes that there is a gradual decrease in K so that the fractional change of the estimated plastic zone sizes (r_p) remains constant with increase in a and that there is no overload effect on crack growth.

The acceptable values of C depend on load ratio, alloy type and environment. If K-increasing and K-decreasing fatigue data agree with each other then the chosen value of C is permitted. This means that C can only be selected from separate experiments if it is not already established for the material to be tested. Moreover, this ASTM proposed method requires very long testing times.

A more efficient decreasing ΔK method is claimed to have been developed by Bailon *et al.*[15] using another load shedding routine in which

$$\Delta P = \Delta P_i \exp(-QN) \quad (3)$$

ΔP and ΔP_i are the instantaneous and initial load, N is the number of elapsed cycles and Q is a damping coefficient given by

$$Q = \frac{1}{r_p}\frac{dr_p}{dN} \quad (4)$$

The basic principle of this method is to approach ΔK_{th} by steps of load shedding according to equation (3) until crack arrests at ΔK_a (which is larger than ΔK_{th} due to overloading effects). The test is resumed with a new set of values for Q (half the previous magnitude) and ΔP_i (and hence ΔK_i, which is half the sum of the original ΔK_i and the associate ΔK_a). Values of ΔK_a which are similar for the last two or three iterative steps indicate that ΔK_{th} has been reached. As opposed to the ASTM method for which dK/da is maintained constant (equation (2)), the Bailon *et al.* method uses decreasing dK/da gradients in the load shedding program. Even though it is said to give 50% better efficiency than the ASTM method this technique does not seem to be appropriate for generating precise data for da/dN below 10^{-8} m cycle^{-1}. In addition it also requires some preliminary tests to determine the best damping coefficient Q.

There are also other variations of the decreasing ΔK method. For example, load shedding can be programmed at either constant stress ratio[16] (R) or increasing stress ratio[16,17]. In the constant R technique ΔK is progressively reduced by steps of 10% or less and the crack is allowed to grow past the plastic zone created by the maximum K of the previous step. ΔK_{th} is considered to have been reached if after the last ΔK step there is no crack growth in a few million cycles. This is a time-consuming process because of unavoidable delay in crack growth due to load history effects. A much faster technique is to use variable R by keeping the maximum K constant and increasing the minimum K in steps. Since there is no overloading effect ΔK_{th} can be obtained most efficiently. However, this increasing R method suffers from the limitation that it is impossible to control the stress ratio at ΔK_{th} and indeed it is difficult to obtain ΔK_{th} for low stress ratios.

A NEW COMPUTER-AIDED METHOD FOR NEAR-THRESHOLD FATIGUE TESTING

In view of the disadvantages apparent in some of the near-threshold fatigue testing methods described in the last section it is our purpose to develop a new computer-aided method which is aimed at overcoming these problems. It is not fully computerized because an automated technique to measure crack length that has adequate accuracy and is suitable for polymers has not yet been developed. Crack lengths are currently measured by a travelling microscope with a resolution of 0.01 mm and input to the computer, which is interfaced with the fatigue machine. The principle of the proposed method is to search for ΔK for a given da/dN by an iteration scheme as shown in *Figure 1*. The technique is somewhat different to the load shedding schedules given by equations (1) and (3) but it leads much faster to ΔK_{th}. The crack tip plastic zone size is calculated according to Dugdale's line zone model[18]. To avoid overloading effects the crack growth (Δa) is allowed to advance twice as long as the plastic zone in the previous ΔK step. Depending on whether the incremental crack growth rate $\Delta a/\Delta N$ is bigger or smaller than the preset da/dN, ΔK is decreased or increased accordingly. At the end of the iterative procedure the current and previous plastic zone sizes become essentially the same, being defined by the 2% tolerance set in the computer program. *Figure 2* shows one of the iterative procedures of the present method in which ΔK changes with the number of elapsed cycles. Here da/dN is preset at 1.4×10^{-9} m cycle^{-1}, $R = 0.2$ and frequency is 5 Hz. ΔK_i is initially chosen with a relatively large value of 0.70 MPa m$^{1/2}$ so that the crack growth is easily initiated and exceeds the preset da/dN value. According to the last step in the flowchart (see *Figure 1*) K_{max} is reduced by half the initial value $K_{max,i}$ and K_{min} is also decreased according to the R ratio, so that the new ΔK is 0.35 MPa m$^{1/2}$. The fatigue test is continued and after $\Delta N = 20\,000$ cycles (which is arbitrarily chosen in this case) the test is stopped

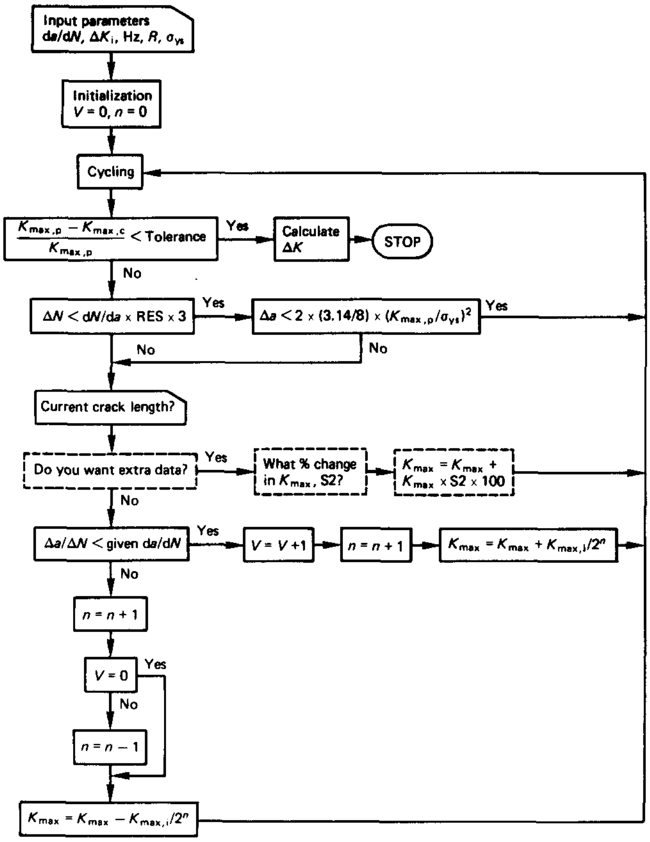

Figure 1 Flowchart for the proposed iteration test method. Subroutine for increasing ΔK test is shown in dashed frames. $K_{max,p}$, $K_{max,c}$ and $K_{max,i}$ refer to the maximum value of K for the immediately preceding, current and initial test conditions; RES is the resolution of the travelling microscope for crack length measurement; S2 is percentage change in K_{max}; R is stress ratio and σ_{ys} is yield strength

and crack growth Δa is measured. (In a fully computerized method it is not necessary to stop the test since Δa is automatically determined and then compared with the plastic zone size r_p.) Since $\Delta a < 2r_p$ and $\Delta a/\Delta N$ is less than the preset value, K_{max} has to be increased by an amount equal to $K_{max,i}/2^n$ where $n=2$. Similarly, K_{min} is increased according to the R ratio. The new ΔK becomes 0.525 MPa m$^{1/2}$ and the fatigue test is continued again as before. After another $\Delta N = 20\,000$ cycles it is found that $\Delta a/\Delta N$ exceeds the preset value so that ΔK has to be reduced accordingly. This gives $\Delta K = 0.35$ MPa m$^{1/2}$. This iterative procedure is repeated with ΔK either increasing or decreasing depending upon whether $\Delta a/\Delta N$ is smaller or larger than the preset value. *Figure 2* shows the successive changes in ΔK with N after the third iteration. Notice how quickly the required ΔK ($=0.25$ MPa m$^{1/2}$) is reached for a given da/dN ($=1.4\times10^{-9}$ m cycle^{-1}) in this example.

In the computer program we have also built in a subroutine to obtain fatigue crack growth data at increasing ΔK steps after the threshold value ΔK_{th} is reached. This provides a check on the data obtained from the main program in which da/dN is gradually reduced. The subroutine is shown in the flowchart of *Figure 1*.

It is relatively easy and economical to implement the proposed computer-aided method. Very simply, the integrated system consists of a fatigue testing machine, a microcomputer, a travelling microscope and interface circuits connecting the fatigue machine and the computer. The interface circuits are needed to record the cycle counter and to control the mean load and amplitude of the cyclic loading. In our own test set-up, the fatigue machine used was a Shimadzu closed-loop hydraulic servo pulser which was equipped with a temperature cabinet and the microcomputer was a Microbee IC with a Z80 CPU, 32K RAM, 16K ROM, 8 bit parallel port, an RS232 serial point and a cassette interface. Near-threshold crack growth was measured by a travelling microscope with a resolution of 0.01 mm. *Figure 3* shows the complete experimental set-up for this work.

EXPERIMENTAL

The unplasticized PVC material was supplied for testing by Vinidex Tubemakers Pty Ltd as 150 mm class 12 pipes. Two levels of processing were obtained in the pipes, the difference being a consequence of the different extruder temperature used. These pipes were thus classified as nominally well and poorly processed. A methylene chloride temperature test according to Van der Heuval[19] measured the temperature at which the first sign of attack occurs on the chamfered surfaces of the uPVC samples and this gave 24 and 2°C for the well and poorly processed pipes, respectively. Pipe lengths of approximately 200 mm were cut and split into two halves giving two samples of about 200 mm by 200 mm. The pipe wall thickness was about 9 mm. These samples were subsequently flattened at about 100°C for 20 min and allowed to cool to room temperature between supporting plates with a moderate pressure. Some residual memory remained in the flattened sheets and they would partially

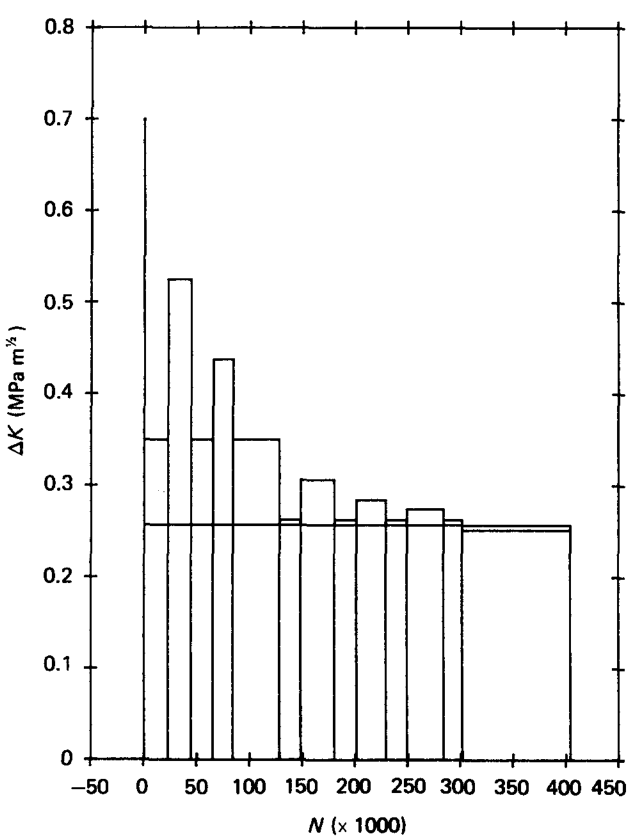

Figure 2 An example of the iterative method showing dependence of ΔK on N; $\Delta K_i = 0.7$ MPa m$^{1/2}$

Figure 3 The complete experimental set-up for the computer-aided test method

return to the pipe shape if heated above the glass transition temperature of $\sim 85°C$.

Fatigue specimens 200 mm by 70 mm were prepared so that the load was always applied perpendicular to the extrusion direction. Near-threshold and high rate fatigue crack growth data were obtained from single-edge-notched (SEN) specimens at 32°C in a Shimadzu servo-pulser and the cyclic load waveform was sinusoidal. The cyclic frequency was kept constant at 5 Hz but the stress ratio (R) defined as K_{min}/K_{max} was varied between 0 and 0.8.

RESULTS AND DISCUSSION

Comparisons of near-threshold fatigue testing methods

Figure 4 shows the fatigue crack growth results for the pipe material over a wide range of growth rates from 10^{-5} to 10^{-10} m cycle^{-1}. The Paris power law relationship: $da/dN = 7.41 \times 10^{-8} \ (\Delta K)^{2.64}$ m cycle^{-1} is valid even down to 10^{-9} m cycle^{-1}. These data were collected by several methods. For $da/dN > 10^{-8}$ m cycle^{-1}, denoted by symbol A in *Figure 4*, the computerized automatic technique based on screen-printed conductive surface grids developed by Mai and Kerr[20] was used. For the near-threshold region whose $da/dN < 10^{-8}$ m cycle^{-1}, the ASTM method was used to obtain data points B and the present proposed technique was employed to derive data points D by the main program and data points C by the increasing ΔK subroutine. In the ASTM load shedding method the constant C of equation (1) was taken as 0.08 mm^{-1}. Since the data points given by B and C fall within the same scatter band it is concluded that overloading effects on da/dN are negligible in this uPVC material if $C = 0.08$ mm^{-1}. We have not, however, tested with other C values which might be more suitable for polymers. In view of the results given in *Figure 4* we conclude that the ASTM method and our proposed method are equally useful for obtaining near-threshold

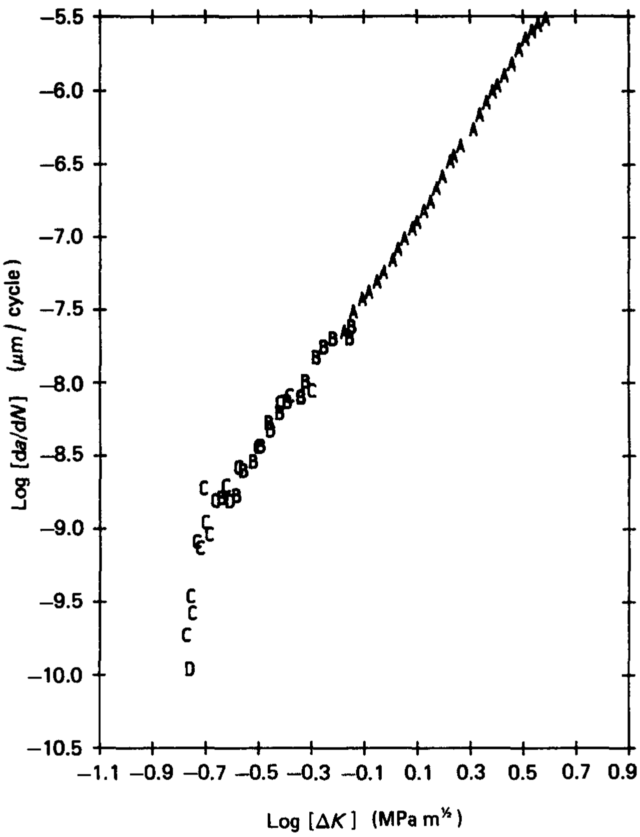

Figure 4 Fatigue crack growth rates for a well processed uPVC pipe material (32°C, 5 Hz, $R=0.2$). For symbols A, B, C see text

Table 1 Comparison of the efficiency of ASTM and present proposed methods for near-threshold crack growth measurement ($R=0.2$, freq. = 5 Hz)

da/dN ($m\,cycle^{-1}$)	ΔK_i ($MPa\,m^{1/2}$)	Tolerance[a] (%)	Duration of test (h) ASTM	Duration of test (h) Proposed method
4.3×10^{-9}	0.7	6.67	53.10	17.34
1.49×10^{-9}	0.7	2.17	160.80	22.45
10^{-10}	0.2	1.8	–[b]	225.66

[a] Tolerance limit set for present proposed test method only. This is not relevant to ASTM method

[b] Not conducted because of unnecessarily long times

fatigue crack growth data. However, a major difference exists in terms of the efficiency of the experimental technique to obtain a given da/dN. *Table 1* compares the testing times required for these two methods to achieve three levels of da/dN, equal to 4.3×10^{-9}, 1.49×10^{-9} and 10^{-10} m cycles^{-1}. For the two larger crack growth rates the present method is about 3–7 times faster than the ASTM method. *Figure 5*, which shows the variation of ΔK with crack length, confirms that our proposed technique is superior to the ASTM method in reaching the specified near-threshold da/dN value. The improvement in testing times is enhanced as the crack growth rate is further reduced. To get down to 10^{-10} m cycle^{-1}, which is generally regarded as the practical threshold crack growth rate[5], our proposed method took about 9.4 days at 5 Hz. We expect that the ASTM method would take considerably longer.

The testing times of the present method may depend on the initial applied ΔK_i because of the crack initiation process. Consequently, a large ΔK_i gives a shorter time for crack initiation and vice versa. But once a crack is initiated ΔK_i does not make much difference to the test duration, provided this time for crack initiation is accounted for. In our experiments, within 18 loading steps, ΔK converges for a given da/dN with less than 2% tolerance, regardless of ΔK_i. We also expect the testing times to depend strongly on the overloading sensitivity of the material.

Stress ratio (R) and processing effects on fatigue threshold

To study the effects of stress ratio (R) and level of processing on fatigue threshold (ΔK_{th}) we have assumed, somewhat arbitrarily, that this threshold value occurs at a crack growth velocity of approximately 3.15×10^{-10} m cycle^{-1}. Judging from the near-threshold fatigue data given in *Figure 4*, this assumption is not unreasonable. We have also chosen a tolerance of $<2\%$ in the test program for fatigue threshold. *Figure 6* shows a plot of ΔK_{th} *versus* stress ratio R and it can be seen that for $0.5<R<1.0$, ΔK_{th} remains constant at $0.15\ MPa\,m^{1/2}$. There are very few models[12,16,21–23] presented for R-ratio effects on ΔK_{th} and most of these cannot be readily applied to polymers. Invariably, all models are associated with some form of crack closure effects. As shown below, the ΔK_{th} data appear to be consistent with the simple mechanistic model suggested by Schmidt and Paris[12]. If it is assumed that there is a constant crack-tip-closure stress-intensity factor K_{cl} independent of R ratio and that for crack growth a constant effective threshold stress intensity factor, ΔK_0, is required, then for low R-ratio cases where $K_{min}\leqslant K_{cl}$, the measured fatigue threshold

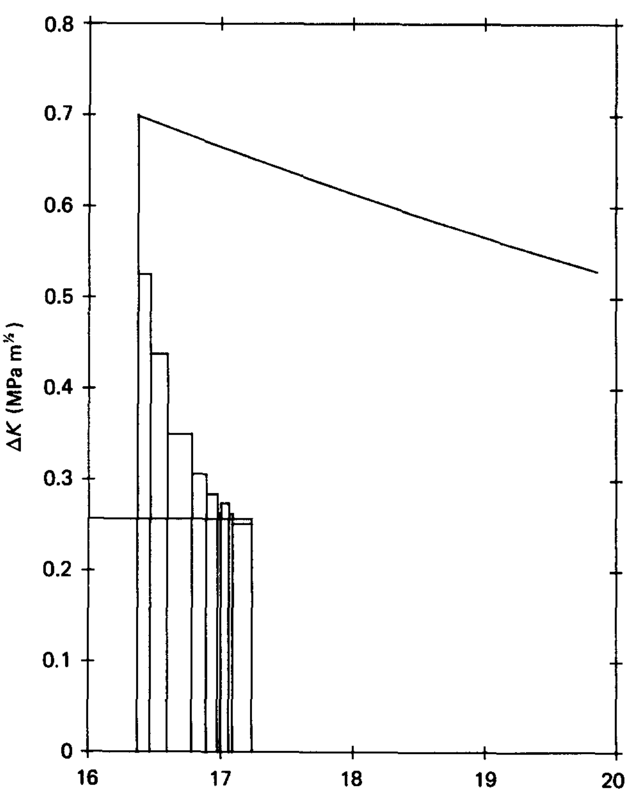

Figure 5 Comparison of the dependence of ΔK on crack length for the ASTM and proposed methods

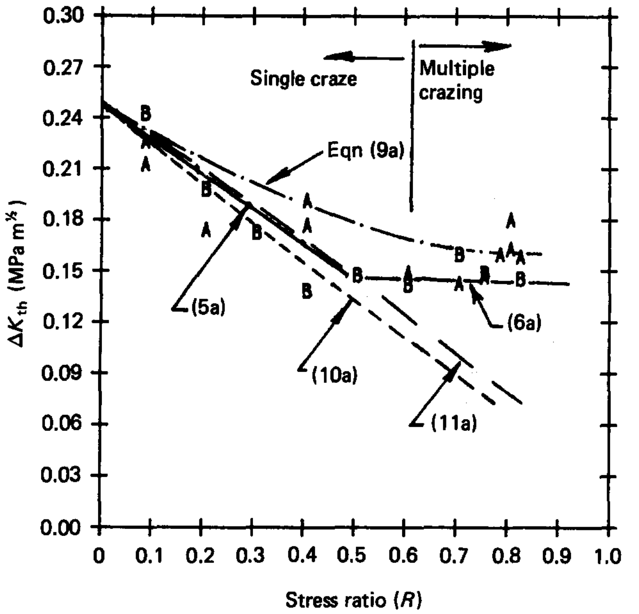

Figure 6 Stress ratio dependence of ΔK_{th} at $da/dN = 3.15\times10^{-10}$, cycle^{-1} for both well processed (A) and poorly processed (B) uPVC pipe material. Theoretical curves are based on Williams' model

ΔK_{th} becomes:

$$\Delta K_{th} = K_{max}(1-R) = (K_{cl} + \Delta K_0)(1-R) \qquad (5a)$$

$$K_{max} = (K_{cl} + \Delta K_0) = \text{constant} \qquad (5b)$$

and for high R-ratio cases, where $K_{min} > K_{cl}$,

$$\Delta K_{th} = \Delta K_0 = \text{constant} \qquad (6a)$$

$$K_{max} = \Delta K_0/(1-R) \qquad (6b)$$

Equations (5a) and (6a) therefore predict two linear relationships between ΔK_{th} and R as shown in *Figure 6*. The transition appears to occur at $R_c = 0.5$, and the effective threshold $\Delta K_0 = 0.15\,\text{MPa m}^{1/2}$ and closure stress intensity $K_{cl} = 0.10\,\text{MPa m}^{1/2}$. K_{cl} obtained here is lower than $K_{cl} = 0.30\,\text{MPa m}^{1/2}$ derived in Part 1 of this series[24], where the test temperature, 20°C, is lower than the 32°C in these experiments. The temperature effect on K_{cl} is expected. *Figure 7* also plots the variation of K_{max} with R at the fatigue threshold. There is good agreement with the predictions based on equations (5b) and (6b) using $K_{cl} = 0.10\,\text{MPa m}^{1/2}$ and $\Delta K_0 = 0.15\,\text{MPa m}^{1/2}$. Despite its apparent success the Schmidt and Paris model bears no relation to the fatigue propagation mechanisms observed in the experiments.

In Williams' model[21] (see also Part 1 of this series)[24] the fatigue threshold ΔK_{th} is given by

$$\Delta K_{th} = K_{max}(1-R) \qquad (7a)$$

where

$$K_{max} = [\alpha K_c^2/F(R) + K_{cl}^2]^{1/2} \qquad (7b)$$

αK_c^2 is the fatigue threshold when $K_{cl} = 0$ and there is complete unloading in each cycle, which is therefore equivalent to ΔK_0; $F(R)$ is a function of the stress ratio depending on the deformation mechanisms taking place at the tips of the crack and craze zone, respectively. For the crack-tip-zone loaded case,

$$F(R) = (1-R)^2 + \alpha R^2 \approx (1-R)^2 \qquad (8a)$$

and for the craze-tip-zone loaded case,

$$F(R) = 1 - (1-\alpha)^2 R^2 \approx 1 - R^2 \qquad (8b)$$

since $\alpha \ll 1.0$. Equation (8a) is relevant to materials which are more ductile, such as PVC, where shear yielding and multiple crazing (or craze bunches) can occur at the crack tip. On the other hand, for materials with a strong propensity for single craze formation, such as PMMA, equation (8b) is more appropriate, since now the load can be sustained at the craze tip region. It will be shown in the next section that for $R > 0.6$ the formation of multiple crazes is obvious at the crack tip, but for $R \leqslant 0.6$, only single crazes are observed. From equations (7) and (8) we can obtain for the multiple crazing case:

$$\Delta K_{th} \approx [\alpha K_c^2 + K_{cl}^2(1-R)^2]^{1/2} \qquad (9a)$$

$$K_{max} \approx [\alpha K_c^2/(1-R)^2 + K_{cl}^2]^{1/2} \qquad (9b)$$

and for the single crazing case:

$$\Delta K_{th} \approx [\alpha K_c^2(1-R)/(1+R) + K_{cl}^2(1-R)^2]^{1/2} \qquad (10a)$$

$$K_{max} \approx [\alpha K_c^2/(1-R^2) + K_{cl}^2]^{1/2} \qquad (10b)$$

If both mechanisms can occur simultaneously, Williams shows that $F(R) \approx (1-R)$ by taking the average of equations (8a) and (8b), so that in this case

$$\Delta K_{th} \approx [\alpha K_c^2(1-R) + K_{cl}^2(1-R)^2]^{1/2} \qquad (11a)$$

$$K_{max} \approx [\alpha K_c^2/(1-R) + K_{cl}^2]^{1/2} \qquad (11b)$$

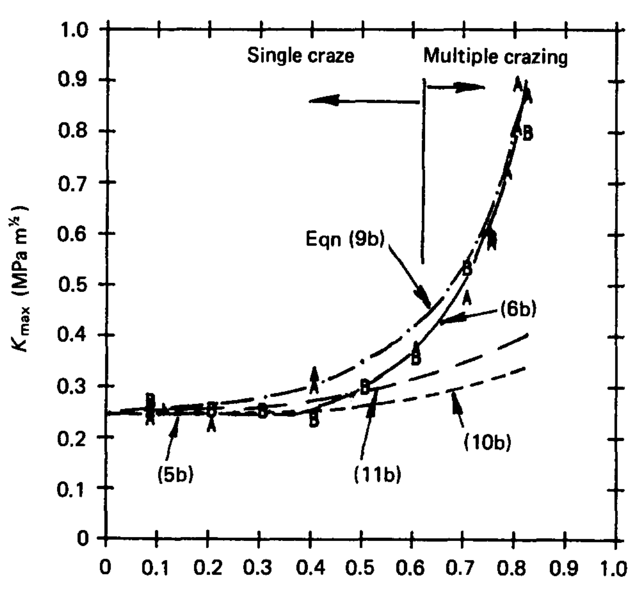

Figure 7 Stress ratio dependence of K_{max} at $da/dN = 3.15\times10^{-10}$m cycle^{-1} for both well processed (A) and poorly processed (B) uPVC pipe material. Theoretical curves are based on Williams' model

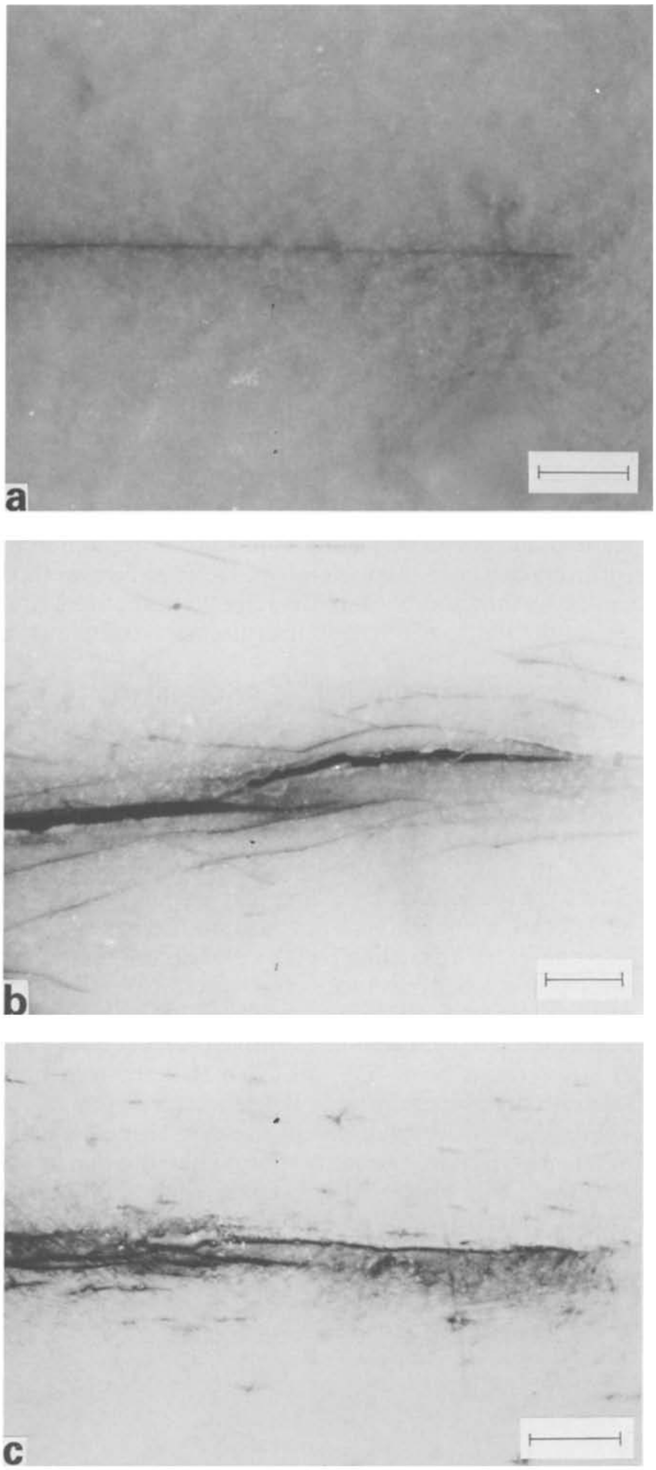

Figure 8 Craze pattern around crack tip at $da/dN = 3.15 \times 10^{-10}$ m cycle^{-1}: (a) poorly processed, $R = 0.08$, $K_{max} = 0.26$ MPa m$^{1/2}$; (b) well processed, $R = 0.8$, $K_{max} = 0.8$ MPa m$^{1/2}$; (c) poorly processed, $R = 0.82$, $K_{max} = 0.79$ MPa m$^{1/2}$. Scale bars represent 20 μm

To see how the Williams model, i.e. equations (9)–(11), can be used to predict ΔK_{th} and K_{max} at the fatigue threshold we need to first estimate αK_c^2 and K_{cl}. When $R = 1.0$, from equation (9a) we obtain $\Delta K_{th} = (\alpha K_c^2)^{1/2} = 0.15$ MPa m$^{1/2}$ from *Figure 6* so that $\alpha K_c^2 = 0.023$ MPa m$^{1/2}$. When $R = 0$, $\Delta K_{th} = (\alpha K_c^2 + K_{cl}^2)^{1/2} = 0.25$ MPa m$^{1/2}$ (from *Figure 6*), which therefore gives $K_{cl} = 0.20$ MPa m$^{1/2}$, which is larger than the 0.10 MPa m$^{1/2}$ estimated from the Schmidt and Paris model using equation (5a). This difference largely arises from the difference between the arithmetic and geometric methods of calculating ΔK_{th}. Also plotted in *Figure 6* are the predicted curves according to equations (9a)–(11a). Clearly, the predicted curves for the single craze mechanism and the combined shear yielding–single crazing mechanism describe the ΔK_{th} data very well for $0 < R \leqslant 0.5$. Good agreement with experimental data for $R > 0.6$ can only be obtained by the predicted curve corresponding to the multiple crazing mechanism. K_{max} at the fatigue threshold is also plotted in *Figure 7* and the same conclusions as those for *Figure 6* can be drawn here. Therefore, provided the exact mechanisms are identified with fatigue crack propagation, the ΔK_{th} and K_{max} threshold data given in *Figures 6* and *7* can be adequately described by Williams' model[21], which is superior to the Schmidt and Paris model since the latter does not include deformation mechanisms in its formulation.

Figure 6 also shows the stress ratio dependence of ΔK_{th} at $da/dN = 3.15 \times 10^{-10}$ m cycle^{-1} for both well and poorly processed uPVC pipes. There is no apparent difference in ΔK_{th}, and hence ΔK_0 and K_{cl}, between these two levels of processing studied. Jilken and Gustafson[4] have shown that, at $R = 0.1$ and 40 Hz, processing temperatures have some effects on ΔK_{th} in a uPVC pipe material. ΔK_{th} is largest at a processing temperature of 194°C and falls on either side of this maximum at other temperatures. Since we have not tested enough materials of different processing levels we may not yet have obtained the maximum ΔK_{th}. The approximately equal ΔK_{th} for the two levels of processing shown in *Figure 6* may just happen to occur on either side of the maximum. Comparison of our ΔK_{th} data at 5 Hz and those of Jilken and Gustafson[4] at 60 Hz shows that increasing the frequency causes an increase in ΔK_{th} for all R ratios. Further discussion on the effects of processing on fatigue crack initiation and propagation will be given in Part 3 of this series.

Craze formation accompanying fatigue crack growth

At the fatigue threshold the formation of crazes at the crack tip region is dependent on both K_{max} and R ratio, but it does not appear to depend on the level of processing of the uPVC pipes. It was mentioned in the last section that for $R < 0.6$ there are no craze bunches or multiple crazing observed at the crack tip – only single crazes can be found. However, when $R > 0.6$ multiple crazes are seen. These two features are clearly demonstrated in *Figure 8*. To confirm this observation further we initially took a poorly processed sample and applied $K_{max} = 0.38$ MPa m$^{1/2}$ and $R = 0.6$ to obtain the fatigue threshold $\Delta K_{th} = 0.15$ MPa m$^{1/2}$. No multiple crazes were seen. As soon as R is increased to 0.85 with K_{max} changed to 1 MPa m$^{1/2}$ multiple crazes happen, as shown in *Figure 9*, after 2000 cycles. The distribution of these crazes is along the direction of the minor principal stress trajectories as discussed by Bevis and Hull[25] (see *Figure 10*). The extent to which crazes can be formed is determined by the major principal stress contour given by[26]:

$$\sigma_p = \frac{K_{max}}{\sqrt{(2\pi r)}} \cos(\theta/2)[1 + \sin(\theta/2)] = \sigma_y \qquad (12)$$

where σ_y is the crazing stress for the polymer. Here $\sigma_p = \sigma_y$

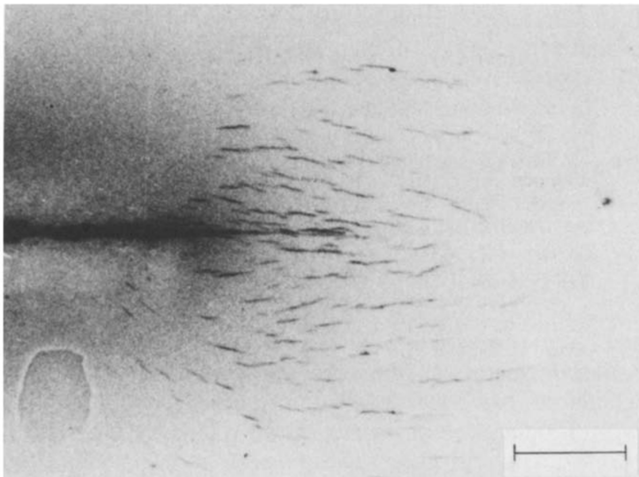

Figure 9 Craze pattern around crack tip in a poorly processed pipe at the threshold that is first subjected to $R=0.6$ and $K_{max}=0.38$ MPa m$^{1/2}$ and then to $R=0.85$ and $K_{max}=1$ MPa m$^{1/2}$. Polarised light used in viewing. Scale bar represents 100 μm

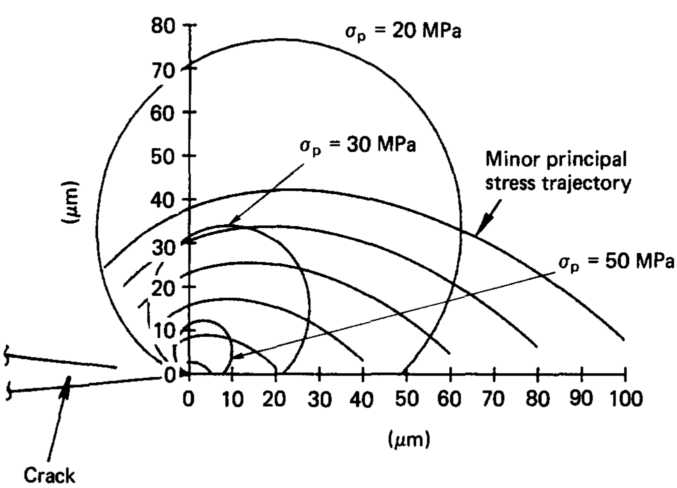

Figure 10 Minor principal stress trajectories shown at crack tip. The maximum principal stress contours for $\sigma_p=50$, 30 and 20 MPa are also shown

is taken as the critical stress criterion for craze formation. *Figure 10* also plots three constant σ_p – loci of 50, 30 and 20 MPa with $K_{max}=0.38$ MPa m$^{1/2}$. For this uPVC material, $\sigma_y=50$ MPa, at the fatigue threshold where $da/dN=3.15\times10^{-10}$ m cycle^{-1} and $R=0.6$, $K_{max}=0.38$ MPa m$^{1/2}$ (from *Figure 7*), we can estimate r_{max} from equation (12) within which crazing can be found. This gives $r_{max}\approx12$ μm. From *Figures 8* and *9* the average vertical spacing between crazes can be seen to be approximately 10 μm for high R ratios. Hence, it is not expected that multiple crazes can be formed at $R<0.6$. Shear yielding can also occur at the crack tip in this material, *Figure 11*, but this deformation mechanism has not been studied in any detail in this work.

A final point that needs to be discussed is the development of the crazes which tend to be aligned in the crack growth direction. That is, as the fatigue crack spreads, the crazes are no longer developed in the direction of the minor principal stress trajectories (see *Figure 9*) but are now parallel to the crack (see *Figures 8b* and *c*). It is difficult to predict exactly the craze growth direction when the fatigue crack also advances. A qualitative description of craze propagation has been given by Bevis and Hull[25], who have assumed that the direction of growth is determined by the superposed minor principal stress trajectories due to the previous and current crack tip positions. In this way, crazes sufficiently away from the initial crack tip tend to be parallel to the propagating fatigue crack. This very simple qualitative analysis does not consider partial unloading effects on the crazes due to crack growth and it is only based on the elastic stress field at the crack tip. We should not, therefore, place too much significance on it.

CONCLUSIONS

A computer-aided method has been successfully developed to measure near-threshold fatigue crack growth rates for a uPVC pipe material. It is shown to be far more efficient than the current ASTM proposed method and it can be readily applied to other materials with very low implementation cost. However, the method can be further improved and made fully computerized if an automatic crack length measurement technique is developed.

The fatigue threshold ΔK_{th} measured at $da/dN=3.15\times10^{-10}$ m cycle^{-1} and 5 Hz decreases with stress ratio up to $R=0.5$ and thereafter remains constant. This R ratio dependence of ΔK_{th} can be accurately predicted using Williams' two-stage line zone model[21] if the deformation mechanisms associated with fatigue crack growth are identified. When $R<0.6$ only single crazes are found. Shear yielding may also have occurred at the crack tip but this has not been studied in any detail.

By considering crazing to take place along minimum principal stress trajectories and a critical maximum principal stress criterion for craze formation, it is shown that at the fatigue threshold multiple crazes cannot form at stress ratios below 0.6, provided that the minimum separation between crazes is not less than 10 μm.

For the two levels of processing investigated for this uPVC material ΔK_{th} remains approximately equal for all R ratios. This implies that even at such a low crack growth rate of 3.15×10^{-10} m cycle^{-1} processing does not seem to affect fatigue crack propagation.

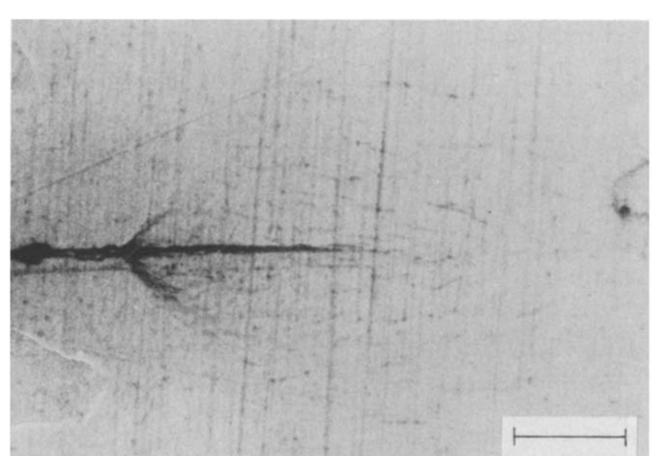

Figure 11 As *Figure 9* but viewed with unpolarized light. Shear deformations observed at $\pm45°$ to the crack line. Scale bar represents 100 μm

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